Why is the Physical World So Comprehensible?

P.C.W. Davies

A vexing scientific mystery of longstanding concerns the peculiar conjunction of simplicity and complexity that pervades the universe. We believe that the underlying laws of physics are simple in form, yet the actual states of the world are highly complex. It is only in recent years that any sort of general understanding of the source of this complexity has emerged.

The most striking feature of many complex systems is their non-random nature. The universe is populated by distinct classes of recognizable *things*: galaxies, stars, crystals, bacteria, people. Given the limitless variety of ways in which matter and energy can arrange themselves, almost all of which would be "random," the fact that the physical world is a coherent collection of mutually tolerant, quasi-stable entities is surely a key scientific fact in need of explanation.

The non-random nature of cosmic complexity is captured by the concept of *organization*, or, to use a more fashionable word, *depth.*[1] According to the best cosmological theories, the universe began in an exceedingly simple state. Indeed the initial state might well have been essentially smooth empty space. It is hard to think of anything more "shallow." All the depth that has arisen in the universe is the result of a sequence of self-organizing and self-complexifying processes that have occurred *since* the initial bang. The epithet "creation" in connection with the big bang seems a serious misnomer, since almost all the creative activity that has generated the richness and variety of the present state occurred after the big bang.[2]

The seemingly unidirectional advance of complex organization, or depth, imposes on the universe an arrow of time, which is related to, but distinct from, that due to the second law of thermodynamics. Some people have perceived an element of paradox in the growth of organization in a universe in which entropy always rises. True, the former arrow does challenge the spirit of the second law, which predicts continual degeneration. But there is no conflict with the letter of the law. Self-organization costs entropy. But whereas entropy is a measure of information loss, organization (or depth) refers instead to the *quality* of information. Entropy and depth are not each other's negatives.

Among the more interesting complex organized systems to have arisen thus is the human brain. Containing as it does an internal representation of the physical world, the brain stands in an unusual relationship with the world. And here the conjunction of simplicity and complexity is inverted: the brain is incredibly complex, but the mental states that it supports make the world seem deceptively simple. We are able to function as human beings because our mental model of the world bestows upon it a coherent unity. When we talk about "understanding" some aspect of nature, we mean slotting the phenomena associated therewith into our existing mental model of "how things are out there."

Paul C.W. Davies is a leading contemporary physicist and author of several books, including God and the New Physics, Superforce*, and* The Cosmic Blueprint. *He is currently Professor of Physics and Mathematical Physics at the University of Adelaide, South Australia. Davies is also a member of the core team for the long-term, interdisciplinary research project co-sponsored by CTNS and the Vatican Observatory.*

Is this process of understanding a surprise? Does it tell us anything significant about the structure of the brain or the world, or both? Many people have puzzled about such issues. Why is the universe knowable? After all, given the enormous complexity and interconnectedness of the physical world, how can we know anything without knowing everything? Indeed, how can we know anything at all?

As a starting point in addressing these tough questions, let us agree at least on the following statements:

A. There exists a real external world which contains certain regularities. These regularities can be understood, at least in part, by a process of rational enquiry called the scientific method.

B. Science is not merely a game or charade. Its results capture, however imperfectly, some aspect of reality. Thus these regularities are *real* properties of the physical universe and not just human inventions or delusions.

"Why is the Physical World so Comprehensible?" was first published in *Complexity, Entropy and the Physics of Information*, Addison-Wesley Press, and we thank both the author and the publisher for their permission to reprint it in the *CTNS Bulletin*.

In making these assumptions one has to eschew extreme idealistic philosophies, such as those in which the mind somehow imposes the regularities on the world in order to make sense of it. Unless one accepts that the regularities are in some sense objectively real, one might as well stop doing science.

As science progresses, so some regularities become systematized as *laws*, or deductions from them. At this epoch the laws found in our textbooks image only imperfectly the actual regularities. Two points of view can be detected among practicing scientists regarding the ontological status of these laws. The first is that there exist "real" laws, or "the correct set" of laws, to which our current theories are only an approximation. As science progresses so we converge upon the "true" laws of the universe, which are regarded as eternal, timeless, and transcendent of the physical states.

By contrast, some scientists deny that there are any "true" laws "out there," existing independently of scientific enquiry. What we call laws, they maintain, are simply our attempts to cope with the world by ordering our experiences in a systematic way. The only laws are *our* laws, and they are to be judged solely on utilitarian grounds, i.e., they are neither true nor false, but merely more or less useful to us. My impression is that many scientists who practice what one might loosely call applied science incline to the latter philosophy, while those engaged in "fundamental" research, for example, on quantum cosmology or the unification program, adopt the former position.

The issue of whether the laws of nature are discovered or invented is sidestepped if we view the world algorithmically.[3] The existence of regularities may be expressed by saying that the world is *algorithmically compressible.*[4] Given some data set, the job of the scientist is to find a suitable compression, which expresses the causal linkages involved. For example, the positions of the planets in the solar system over some interval constitute a compressible data set, because Newton's laws may be used to link these positions at all times to the positions (and velocities) at some initial time. In this case Newton's laws supply the necessary algorithm to achieve the compression.

Viewed this way, the question "Why is the universe knowable?" reduces to "Why is the universe algorithmically compressible?" and "Why are human beings so adept at discovering the compressions?"

First it should be noted that our mental model of the world is itself an algorithmic compression. If the world were not compressible in this way, there could be no cognition. So the very fact that we exist as observers already constrains the universe to have the property of algorithmic compressibility. Of course, this anthropic reasoning does not constitute an *explanation* of why the universe is compressible; it merely tells us that we could not be around to debate the issue were it not so. (There is another anthropic connection here. The existence of biological organisms implies depth. But an algorithmically incompressible universe would necessarily be shallow.)

Given the limitless variety of ways in which matter and energy can arrange themselves, almost all of which would be "random," the fact that the physical world is a coherent collection of mutually tolerant, quasi-stable entities is surely a key scientific fact in need of explanation.

Secondly, there is a wide class of physical systems, the so-called chaotic ones, which are *not* algorithmically compressible.[5] One can imagine a universe in which there are no regularities at all, only chaos. The fact that there is cosmos rather than chaos is the starting point of science. The existence of non-chaotic dynamical regimes is a profound fact about nature, and one can ask for an explanation of this fact. I don't know whether we will ever have such an explanation, or what sort of explanation it might be. But one possible strand of reasoning might run thus: The non-chaotic nature of many systems seems to hinge on their approximate linearity and this, in turn, depends on the smallness of certain coupling constants, radiative corrections, etc. At present we do not have a theory of coupling constants, but one may emerge from attempts at grand, or super, unification. If a satisfactory unified theory is found for which these constants are fixed, then that will constitute a partial answer as to why chaos is kept at bay.

Merely avoiding chaos in the technical sense is a necessary, but not sufficient, condition for *practical* algorithmic compression. One can imagine a world in which the relevant algorithms are impenetrably complicated, too complicated to be discovered by systematic enquiry, or even too complicated to be tested in the age of the universe by any conceivable computational system. Part of the reason for the apparent simplicity of the laws of physics rests with the key property of *locality*. If everything in the universe interacted with everything else in a highly non-local manner, we could not untangle all the components to discover the algorithms. That is, we could never know something without knowing everything.

In fact, the study of quantum cosmology compels us to address the question of how quasi-locality *has* been established in the universe.[6] One expects the quantum state of the universe which emerged from the Planck era to describe unlocalized matter and energy. The quasi-locality that we now observe, in which material objects and the cosmological scale factor are described by sharply peaked wave packets is, according to some, an extremely unusual state. The explanation for this state is evidently to be sought by appeal to a very special initial quantum state of the universe, combined with arguments about environmental decoherence.[7] The point that I wish to make here is that algorithmic compressibility in principle is one thing, but our ability to actually effect the compression is quite another, and likely to be possible only in a universe in which strong constraints are placed on, for example, the initial quantum state. Without special choice of quantum state, it is very likely that we could *not* know anything without knowing everything.

The issue of whether the laws of nature are discovered or invented is sidestepped if we view the world algorithmically. The existence of regularities may be expressed by saying that the world is algorithmically compressible.

This brings me to the question of why, even in a quasi-local universe, humans are so good at discovering nature's algorithms. I consider this to be a sharper version of Wigner's famous question about the "unreasonable effectiveness" of mathematics in the physical sciences.[8] Mathematics emerges from the highest form of human mental activity (which is possibly the deepest known complex process) yet it finds ready application to the external world at its most basic level. More specifically, the fundamental laws of physics seem to be expressible as succinct mathematical statements. But these statements are precisely the algorithms which compress the data of experience. Hence we are really dealing with our "unreasonable" ability to spot those algorithms. Again, does this fact tell us something important about the structure of the brain, or the physical world, or both?

Let me express this point in a somewhat novel way. Hawking has claimed that "the end of theoretical physics may be in sight."[9] He refers to the promising progress made in unification, and the possibility that a "theory of everything" might be around the corner. Although many physicists flatly reject this, it may nevertheless be correct. As Feynman has remarked, we cannot go on making discoveries in physics at the present rate for ever. Either the subject will bog down in seemingly limitless complexity and/or difficulty, or it will be completed.[10]

Suppose that the latter optimistic view is correct, and suppose further that the superstring theory or something like it emerges in a few decades as a satisfactory "theory of everything."[11] Then it will be the case that a very limited period mathematical development (300 or 3000 years, depending on where you start) will have proved sufficient to encapsulate the ultimate laws of the cosmos. But this raises the curious question of why such a glittering prize, so sweeping in its explanatory power, demands a *nontrivial*, yet so astonishingly *limited*, amount of mathematics.

One can imagine a world in which the principles are transparent to us all at a glance, or another world in which the principles are impenetrably complicated and subtle. Given the limitless amount of mathematics which could (and maybe will) be developed in the (possibly infinite) future, isn't it remarkable that one could have all of fundamental physics wrapped up with so modest a mathematical investment? Given that the world does require some subtle and sophisticated mathematics to describe it, why is it (relatively!) so *easy* for us to achieve this unifying description?

There is another aspect to this point. Again, assuming a "theory of everything" is within our grasp, why is it that the requisite mathematics is achievable by the (severely limited) human brain using an education span that is less than a typical human life span? I confess I find this exceedingly odd. The learning capabilities of the brain, and the length of the human life span, are both dictated by Darwinian criteria, and (presumably) have no connection whatever with the mathematical form of the fundamental laws of the cosmos.

. . . algorithmic compressibility in principle is one thing, but our ability to actually effect the compression is quite another, and likely to be possible only in a universe in which strong constraints are placed on, for example, the initial quantum state.

It is often said that, because the brain is a physical system (i.e., part of the physical world), it is no surprise that it reflects so efficiently the workings of that world, i.e., that it generates just that mathematics which express the very law of physics that govern its own activity. I consider this to be an entirely erroneous argument, based on a confusion of conceptual levels (a muddle between hardware and software). As I have discussed this in detail elsewhere,[12] I shall here restrict myself to a new development that has a bearing on this issue, namely, the question of computability in physical law.

Most mathematicians subscribe to the so-called Church-Turing hypothesis, which is to say that a Turing machine, or universal computer, can perform any computable mathematical operation. In other words, if a mathematical problem is solvable, a Turing machine can solve it (so long as there is no restriction on the available memory storage space). This is usually regarded as telling us something about the foundations of mathematics or logic, but as David Deutsch has pointed out, it also tells us something about the physical world.[13] To perform its modest repertoire of operations, a Turing machine must employ the laws of mechanics. If the laws of the physical universe were very different, then some operations that are computable in our universe might no longer be. Conversely, certain operations which are non-computable in our universe might be computable in a hypothetically universe with different laws.

Deutsch expresses it thus:[14]

> The reason why we find it possible to construct, say, electronic calculators, and indeed why we can perform mental arithmetic, cannot be found in mathematics or logic. *The reason is that the laws of physics 'happen to' permit the existence of physical models for the operations of arithmetic* such as addition, subtraction and multiplication. If they did not, these familiar operations would be non-computable functions.

We are so used to the fact that simple arithmetic *works* in daily life that we take its efficacy completely for granted. Yet the world does not have to be that way. The mathematician R.W. Hamming writes:[15]

> I have tried, with little success, to get some of my friends to understand my amazement that the abstraction of integers for counting is both possible and useful. Is it not remarkable that 6 sheep plus 7 sheep make 13 sheep; that 6 stones plus 7 stones make 13 stones? Is it not a miracle that the universe is so constructed that such a simple abstraction as a number is possible? To me this is one of the strongest examples of the unreasonable effectiveness of mathematics. Indeed, I find it both strange and unexplainable.

We seem to have encountered a logical loop here. The laws of physics define the allowed mechanical operations that occur in the physical universe, and thence the possible activities of a Turing machine. These mechanical operations thus determine which mathematical operations are computable and define for us what might be called simple solvable mathematics (like addition). For some reason, those same laws of physics can be expressed in terms of this simple mathematics. There is thus a self-consistency in that the laws generate the very mathematics that makes those laws both computable and simple.

One can now ask whether this is the *only* self-consistent loop. Is it possible that there could exist a world with very different laws, in which, say, arithmetic could not be performed, but in which some other set of mathematical operations, non-computable in our world, were not only computable, but also described those different laws in simple terms?

The fundamental laws of physics seem to be expressible as succinct mathematical statements. But these statements are precisely the algorithms which compress the data of experience. . . . does this fact tell us something important about the structure of the brain, or the physical world, or both?

If there is only one self-consistent loop, then it implies there is only one possible computable universe. But, suppose that other loops are possible. What, if anything, is special about our universe? Might ours be the only universe that is both computable and cognizable? (Cognizability might demand both computability and depth, i.e., a certain level of organized complexity.) Alternatively, our universe might represent *maximum potential variety* in some sense. Smolin[16] has suggested that the logical structure of the universe might be such as to generate the richest variety of organized forms, echoing Leibniz's "best of all possible worlds." Yet another possibility, suggested by Barrow,[17] is that the laws of physics might be "optimally encoded" in the Shannon information-theoretic sense, rendering them relatively robust to the filter of observation.

All these speculations hinge on the assumption that the laws of physics do indeed involve computable mathematics, and that the brain is a Turing machine. But some writers have questioned both these assumptions. Geroch and Hartle[18] have investigated the possibility that at least some physical processes might involve non-computable mathematical descriptions. They point out that certain procedures for calculating path integrals in quantum gravity involve non-computable sums over topologies. Recently, Penrose[19] has suggested that the human brain has capabilities over and above those of a Turing machine, because humans are able to discover the existence of true mathematical statements that no Turing machine can prove. He claims that this ability can be traced to the influence of quantum mechanics on brain processes. (A conventional Turing machine is a classical system.) If either of these conjectures is correct, it would add a subtle new twist to the question of why the universe is comprehensible to us.

There is a further tacit assumption running through all these arguments, which is that the laws of physics are timeless eternal truths. But the intimate relationship between physics and computation which is emerging from such studies challenges that assumption. If nature can be viewed as a computational process ("the universe is a computer" according to Fredkin[20]), then the form of the physical laws might be constrained by what can be in principle computed. This point has been made by Landauer.[21] One then has to address the question of the *computational limits* of the cosmos. If something cannot be computed by the entire universe during the age of the universe, in what sense can it be said to be computable?

. . . suppose further that the superstring theory or something like it emerges in a few decades as a satisfactory "theory of everything.". . . this raises the curious question of why such a glittering prize, so sweeping in its explanatory power, demands a nontrivial, yet so astonishingly limited, amount of mathematics.

Might this imply that the laws of physics somehow "fade away" as one goes back towards the initial singularity, on account of the fact that the computational power of the universe tends to zero as $t \to 0$? Such a possibility has been suggested by Lloyd and Pagels.[22] If so, then the laws of physics, along with the state of the universe, would evolve with time. The laws would somehow emerge from the big bang, and gradually "congeal" into their timeless form. Such a speculation is not new, of course; one of its more eloquent proponents is John Wheeler.[23]

To make this more concrete, I should like to point out that one may obtain a natural measure of the information capacity of the cosmos using the Hawking-Bekenstein formula for black hole entropy.[24] If the entire universe were converted into a black hole, it would conceal a quantity of information I_u given by

$$I_u \quad GM_u^2 / hc$$

where M_u is the mass of the observable universe (i.e., within the particle horizon). At the current epoch t_0, $I_u \quad 10^{120}$. At epoch t

$$I_u(t) \quad 10^{120}(t / t_0)^2$$

so that at the Planck time, $t_P \quad 10^{-43}s$, $I_u \quad 1$, as expected. The Bekenstein-Hawking information is the maximum possible information capacity for a system of mass M_u.

The above formula places a bound on the cosmological equivalent of the amount of "blank tape" available to a Turing machine and suggests that, at the Planck epoch t_P, effective computation must cease. Mathematical operations associated with laws of physics would not be implementable at all. Does this imply that the laws of physics are meaningful only for t » t_P?

The memory capacity of the universe is only part of the story. There will be a further bound due to the finite information *processing rate*. This will be somewhat model-dependent. In some cases, such as certain matter-filled Friedmann models, the computational power goes *up* as the singularity is approached. (Barrow and Tipler have made a study of this.[25]) On the other hand, a universe which starts out as a gravitationally smooth vacuum would seem unpromising as a computational device. Evidently some universe models might admit sharply defined laws at all times, others not. It should be emphasized that models which require a "law of initial conditions" (e.g., Hartle-Hawking[26]) made no sense except in the context of timeless eternal laws.

On the philosophical side, there is an urgent need for these speculations to be placed in the context of a theory of mathematics. For example, Platonists believe that mathematics enjoys a timeless, independent existence. It is already "out there," and mathematicians merely discover it. This has been the position adopted by Gödel, Penrose, and many theoretical physicists who work on fundamental problems. Thus in quantum cosmology, for example, one writes down the Wheeler-DeWitt differential equation or the Hartle-Hawking path integral without questioning whether the mathematical operation involved can actually be performed; mathematics is assumed to "already exist." Furthermore, it is often speculated that there exist as-yet-undiscovered laws that involve mathematics which has also yet to be "discovered." One often hears this sentiment expressed in connection with the superstring theory.[27]

By contrast, others (including many computer scientists) reject Platonism and subscribe instead to formalism. According to this philosophical position, mathematics is not discovered; it is invented by mathematicians. Mathematical operations consist of nothing more or less than mappings of one set of symbols into another; in computer parlance, mapping bit strings into bit strings. The idea that there exist, timelessly, physical laws expressed in terms of unknown, and possibly uncomputable, mathematics, is rejected as meaningless.

It has to be concluded that, at this time, the answer to the question "Why is the universe knowable?" is unknown. The dazzling power of mathematics to describe the world at a basic level continues to baffle us. The ability of a subset of the universe (the brain) to construct an internal representation of the whole, including an understanding of that basic level, remains an enigma. Yet a comparison of computational and natural complexity surely provides a clear signpost for the elucidation of these age-old mysteries.

> ***The dazzling power of mathematics to describe the world at a basic level continues to baffle us. The ability of a subset of the universe (the brain) to construct an internal representation of the whole . . . remains an enigma.***

NOTES

[1] Bennett, C.H. "On the Nature and Origin of Complexity in Discrete, Homogeneous, Locally Interacting Systems." *Found. Phys.* 16 (1986):585, and Lloyd, S., and H. Pagels. "Complexity as Thermodynamic Depth." Rockefeller University preprint, 1989.

[2] For a nonspecialist review, see Davies, P.C.W. *The Cosmic Blueprint.* London/New York: Heinemann/Simon & Schuster, 1988.

[3] Chaitin, G.J. *Algorithmic Information Theory*. Cambridge: Cambridge University Press, 1987.

[4] Solomonoff, R.J. "A Formal Theory of Inductive Inference, Part I. *Infor. & Control* 7 (1964):1.

[5] See, for example, Ford, J. "What is Chaos, that We Should Be Mindful of It." *The New Physics*. edited by P.C.W. Davies. Cambridge: Cambridge University Press, 1989, 348, for a discussion of this point.

[6] Halliwell, J.J. "Information Dissipation in Quantum Cosmology and the Emergence of Classical Spacetime," this volume, and the reviews cited therein.

[7] Hartle, J.B. "Excess Baggage." Talk given at the 60th Birthday Celebration for Murray Gell-Mann. Pasadena. January 27, 1989.

[8] Wigner, E. "The Unreasonable Effectiveness of Mathematics in the Natural Sciences." *Comm. Pure Appl. Math.* 13 (1960):1.

[9] Hawking, S.W. "Is the End in Sight for Theoretical Physics?" Inaugural Lecture for the Lucasian Chair, University of Cambridge, 1979.

[10] Feynman, R.P. *The Character of Physical Law*. London: BBC Publications, 1965, 172.

[11] Davies, P.C.W., and J.R. Brown, eds. *Superstrings: A Theory of Everything?* Cambridge: Cambridge University Press, 1988.

[12] Davies, P.C.W. "Why is the Universe Knowable?" *Science and Mathematics,* edited by R. Mickens. Singapore: World Scientific, 1989.

[13] Deutsch, D. "Quantum Theory, the Church-Turing Principle and the Universal Quantum Computer." *Prox. Royal Soc. Lond. A* 400 (1985):97.

[14] *Ibid.*

[15] Hamming. "The Unreasonable Effectiveness of Mathematics." *Amer. Math. Monthly* 87 (1980):81.

[16] Smolin, L. "Space and Time in the Quantum Universe." *Conceptual Problems in Quantum Gravity,* edited by A. Ashtekar and J. Stachel. Boston: Birkhäuser, 1989, and Barbour, J.B. "Maximal Variety as a New Fundamental Principle of Dynamics. *Found. Phys.* 19 (1989):1051.

[17] Barrow, J.D. *The World Within the World*. Oxford: Oxford University Press, 1988, 292.

[18] Geroch, R., and J.B. Hartle. "Computability and Physical Theories." *Found. Phys.* 16 (1986).

[19] Penrose, R. *The Emperor's New Mind.* Oxford: Oxford University Press, 1989.

[20] Fredkin, E. Personal communication, 1989.

[21] Landauer, R. "Wanted: A Physically Possible Theory of Physics." *IEEE Spectrum* 4 (1967):105.

[22] See final section of Lloyd and Pagels, *op. cit.*

[23] See, for example, Wheeler, J.A. "On recognizing `Law without Law'" *Amer. J. Phys.* 51 (1983):398.

[24] Bekenstein, J.D. "Black Holes and Entropy." *Phys. Rev. D.* 7 (1973):2333, and Hawking, S.W. "Particle Creation by Black Holes." *Commun. Math. Phys.* 43 (1975):199.

[25] Barrow, J.D., and F.J. Tipler. *The Anthropic Cosmological Principle*. Oxford: Oxford University Press, 1986, section 10.6.

[26] Hartle, J.B., and S.W. Hawking. "The Wave Function of the Universe." *Phys. Rev. D* 28 (1983) 2960.

[27] See Davies and Brown, *Superstrings, ibid.*